APPENDIX III: Geodesics in Schwarzschild's 4-Space

Schwarzschild's solution to Einstein's equations of gravity, is given by the following expression for the element of arc length:

Eq 46: $$ds^2 = -\gamma^{-1}(dr)^2 - r^2(d\theta)^2 - r^2\sin^2\theta(d\varphi)^2 + c^2\gamma(dt)^2$$

where c is the speed of light and where $\gamma = 1 - \frac{r_g}{r}$, and where $r_g = \frac{2Gm_1}{c^2}$. The variable, m_1, in the expression for r_g, is the sun's mass.

Derivations for Eq 46 were studied in references [2, pp 298-304] and [4, pp ---]. The expression for r_g was obtained from [4, p ---]. These studies were done over a period of about two months, and had as prerequisite the chapter on tensor calculus in [2]. This is a subset of the prerequisites which have already been mentioned in the paragraph following Eq 2, and which provide a foundation for work in many areas of science and engineering.

Eq 46 defines distance in a four dimensional space which is covered by the variables $x^i \leftrightarrow r\ \theta\ \varphi\ t$, where i takes the values 0, 1, 2, 3, and where the superscripted x's stand respectively for r, θ, φ, and t. Thus, the metric tensor components are as shown in Eq 47. If the mass, m_1, is reduced to zero, then the upper left 3 by 3 submatrix is the metric for Euclidean 3-space covered by spherical polar coordinates.

Eq 47: $$g_{ij} : \begin{pmatrix} -\frac{1}{\gamma} & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & c^2\gamma \end{pmatrix}$$

To find mercury's orbit, it is again necessary to solve the equations for a geodesic. Much of the procedure is almost identical with that followed in appendix I from Eq 6 through Eq 17. The Christoffel symbols are to be calculated using the definitions shown in equations Eq 16 and Eq 17, with h replaced by g, and using Eq 47. The resulting non-zero symbols are shown below. The coordinate variables r and θ have been replaced by y_0 and y_1 just as they were in appendix I.

Eq 48: $$\begin{Bmatrix} 0 \\ 00 \end{Bmatrix} = -\frac{r_g}{2\gamma(y_0)^2}$$ **Eq 49:** $$\begin{Bmatrix} 0 \\ 11 \end{Bmatrix} = -y_0\gamma$$