APPENDIX II: Change the Orbit

Make the Orbit More Eccentric

In preparation for illustrating the perihelion shift, it was desirable to increase the eccentricity, e. The first guess made, which worked, was to make E closer to zero. However, in order to compare the new and original orbits, it was also required that the perihelion, r_p, be the same for both orbits. In the derivation which follows the subscript zero refers to the original parameters, while the corresponding quantities without subscripts refer to the new values. For the perihelion to remain unchanged, the following equation must be true:

Eq 38: $$r_p = -\frac{1}{2}\frac{k_0}{E_0}(1-e_0) = -\frac{1}{2}\frac{k}{E}(1-e)$$

Thus:

Eq 39: $$e = 1 - \frac{k_0}{k}\frac{E}{E_0}(1-e_0)$$

But:

Eq 40: $$e = \sqrt{1+\frac{2l^2E}{mk^2}}$$

Solving equations Eq 39 and Eq 40 for l^2 leads the following equation:

Eq 41: $$l^2 = \frac{1}{2}mk_0k\left(\frac{1-e_0}{-E_0}\right)\left[2-\frac{E}{E_0}(1-e_0)\right]$$

If the mass is not changed, so that k_0 and k are the same, then the new value for the angular momentum will be given by the following equation:

Eq 42: $$l = \left\{\frac{1}{2}m_0k_0^2\left(\frac{1-e_0}{-E_0}\right)\left[2-\frac{E}{E_0}(1-e_0)\right]\right\}^{1/2}$$

Eq 42 is used in the worksheet.