Finally, there remains the problem of estimating dS for the numerical solution of the set of differential equations Eq 10 through Eq 15. This is done using the following equations:

$$\left(\frac{ds}{dt}\right)^2 = v^2 = \frac{2T}{m}$$

Eq 35: $$\Rightarrow \quad dt = \sqrt{\frac{m}{2T}}ds = \sqrt{\frac{m}{2T}}\frac{1}{\sqrt{2m(E-V)}}dS = \frac{1}{2T}dS$$

$$\Rightarrow \quad dS = 2Tdt$$

In Eq 35, dt is to be replaced by the period, P, of mercury's orbit divided by the desired number of steps to be taken. In the worksheet, the variable *NR3step* is used for the number of steps. When the worksheet was first being implemented, it was desired to plot half of one orbit, and therefore a factor of two was included in the equations as indicated below. The kinetic energy, T, is replaced by its initial value.

Eq 36: $$2T_0 dt = mr_p^2\dot{\varphi}_0^2 dt = mr_p^2\left(\frac{l}{mr_p^2}\right)^2 dt = \frac{l^2}{mr_p^2}dt$$

Eq 37: $$dS = \frac{l^2}{mr_p^2}\frac{P}{2NR3step}$$

Although Eq 37 uses at value of dt which corresponds to half of the period divided into *NR3step* intervals, taking *NR3step* steps of size dS will not correspond in general to half of the orbit. Thus, by trial and error, the two was changed to three.

This concludes the presentation of mathematics for the section of the worksheet with the title *The Orbit as a Geodesic in a Riemannian 3-space*.