

Now use equations Eq 13 through Eq 15, along with the last part of Eq 8, to define the three functions $f_3, f_4,$ and f_5 . Including only the non-zero Christoffel symbols gives the following three equations:

$$\text{Eq 26:} \quad f_3 = -\left\{ \begin{matrix} 0 \\ 00 \end{matrix} \right\} (y_3)^2 - \left\{ \begin{matrix} 0 \\ 11 \end{matrix} \right\} (y_4)^2 - \left\{ \begin{matrix} 0 \\ 22 \end{matrix} \right\} (y_5)^2$$

$$\text{Eq 27:} \quad f_4 = -2 \left\{ \begin{matrix} 1 \\ 01 \end{matrix} \right\} y_3 y_4 - \left\{ \begin{matrix} 1 \\ 22 \end{matrix} \right\} (y_5)^2$$

$$\text{Eq 28:} \quad f_5 = -2 \left\{ \begin{matrix} 2 \\ 02 \end{matrix} \right\} y_3 y_5 - 2 \left\{ \begin{matrix} 2 \\ 12 \end{matrix} \right\} y_4 y_5$$

The initial conditions are shown in Eq 29. The initial value for r is the perihelion radius, r_p . At perihelion, the time rate of change of r , $\dot{r}_0 \equiv \left[\frac{dr}{dt} \right]_{t=0}$, is zero. For motion in the xy -plane (see the discussion following Eq 4), the initial values of θ and its time derivative will be $\pi/2$, and zero respectively. The value of zero for the initial $\dot{\varphi}$ corresponds to an initial position on the x -axis. The value given for φ , $\dot{\varphi}_0 \equiv \left[\frac{d\varphi}{dt} \right]_{t=0}$, follows from Eq 3.

$$\text{Eq 29:} \quad \begin{bmatrix} r_0 \\ \theta_0 \\ \varphi_0 \\ \dot{r}_0 \\ \dot{\theta}_0 \\ \dot{\varphi}_0 \end{bmatrix} = \begin{bmatrix} r_p \\ \pi/2 \\ 0 \\ 0 \\ 0 \\ l/mr_p^2 \end{bmatrix}$$

The time derivatives in Eq 29 must now be converted into derivatives with respect to the arc length parameter, S , in the Riemannian 3-space. Begin with the expression for arc length, s , in Euclidean three space, and solve for the speed, v , as shown in Eq 30. This is the speed of the planet mercury, which is being treated as a point mass.