

Next, Eq 6 and Eq 7 will be put into the standard form for a system of first order differential equations. In Eq 9,  $y$  and  $f$  are arrays as shown in the next set of equations.

$$\text{Eq 9:} \quad \frac{dy}{dS} = f(y, S).$$

For the Riemannian 3-space this becomes

$$\text{Eq 10:} \quad \frac{dy_0}{dS} = \frac{dx^0}{dS} = \lambda^0 = y_3 = f_0(y_0, \dots, y_5, S)$$

$$\text{Eq 11:} \quad \frac{dy_1}{dS} = \frac{dx^1}{dS} = \lambda^1 = y_4 = f_1(y_0, \dots, y_5, S)$$

$$\text{Eq 12:} \quad \frac{dy_2}{dS} = \frac{dx^2}{dS} = \lambda^2 = y_5 = f_2(y_0, \dots, y_5, S)$$

$$\text{Eq 13:} \quad \frac{dy_3}{dS} = \frac{d\lambda^0}{dS} = -\left\{ \begin{matrix} 0 \\ \alpha\beta \end{matrix} \right\} \lambda^\alpha \lambda^\beta - \left\{ \begin{matrix} 0 \\ \alpha\beta \end{matrix} \right\} y_{3+\alpha} y_{3+\beta} = f_3(y_0, \dots, y_5, S)$$

$$\text{Eq 14:} \quad \frac{dy_4}{dS} = \frac{d\lambda^1}{dS} = -\left\{ \begin{matrix} 1 \\ \alpha\beta \end{matrix} \right\} \lambda^\alpha \lambda^\beta - \left\{ \begin{matrix} 1 \\ \alpha\beta \end{matrix} \right\} y_{3+\alpha} y_{3+\beta} = f_4(y_0, \dots, y_5, S)$$

$$\text{Eq 15:} \quad \frac{dy_5}{dS} = \frac{d\lambda^2}{dS} = -\left\{ \begin{matrix} 2 \\ \alpha\beta \end{matrix} \right\} \lambda^\alpha \lambda^\beta - \left\{ \begin{matrix} 2 \\ \alpha\beta \end{matrix} \right\} y_{3+\alpha} y_{3+\beta} = f_5(y_0, \dots, y_5, S)$$

There are six unknown functions of  $S$ :  $y_0, y_1, y_2, y_3, y_4, y_5$ . The first three of these are the coordinate variables  $x^i \leftrightarrow r\theta\varphi$  for  $i = 0, 1$ , and  $2$ . The last three are the first derivatives of the  $x^i$  with respect to  $S$ . Thus, the first three functions,  $f_i$ , are the last three unknowns, and the last three functions,  $f_{3+i}$ , will be obtained from the last part of Eq 8. First, however, the Christoffel symbols must be defined.