

APPENDIX I: Geodesics in a Riemannian 3-Space

By Eq 1, the metric tensor components are as follows:

Eq 5:
$$h_{ij} = 2m(E - V)g_{ij}$$

where the g_{ij} are given in Eq 2.

The general equations for geodesics in a Riemannian 3-space are given in the next two equations. The Christoffel symbols--the expression in curly braces--will be defined later in the presentation. The summation convention--that repeated indices are summed--is used in Eq 7 and illustrated in Eq 8. The last portion of Eq 8 uses the fact that the Christoffel symbols are symmetric in the lower index pair.

Eq 6:
$$\lambda^i = \frac{dx^i}{dS}$$

Eq 7:
$$\frac{d\lambda^i}{dS} + \left\{ \begin{matrix} i \\ \alpha\beta \end{matrix} \right\} \lambda^\alpha \lambda^\beta = 0 \quad ,$$

Expanding the portion of Eq 7 which contains the Christoffel symbols gives

Eq 8:

$$\begin{aligned} \left\{ \begin{matrix} i \\ \alpha\beta \end{matrix} \right\} \lambda^\alpha \lambda^\beta &= \left\{ \begin{matrix} i \\ \alpha 0 \end{matrix} \right\} \lambda^\alpha \lambda^0 + \left\{ \begin{matrix} i \\ \alpha 1 \end{matrix} \right\} \lambda^\alpha \lambda^1 + \left\{ \begin{matrix} i \\ \alpha 2 \end{matrix} \right\} \lambda^\alpha \lambda^2 \\ &= \left\{ \begin{matrix} i \\ 00 \end{matrix} \right\} (\lambda^0)^2 + \left\{ \begin{matrix} i \\ 10 \end{matrix} \right\} \lambda^1 \lambda^0 + \left\{ \begin{matrix} i \\ 20 \end{matrix} \right\} \lambda^2 \lambda^0 \\ &+ \left\{ \begin{matrix} i \\ 01 \end{matrix} \right\} \lambda^0 \lambda^1 + \left\{ \begin{matrix} i \\ 11 \end{matrix} \right\} (\lambda^1)^2 + \left\{ \begin{matrix} i \\ 21 \end{matrix} \right\} \lambda^2 \lambda^1 \\ &+ \left\{ \begin{matrix} i \\ 02 \end{matrix} \right\} \lambda^0 \lambda^2 + \left\{ \begin{matrix} i \\ 12 \end{matrix} \right\} \lambda^1 \lambda^2 + \left\{ \begin{matrix} i \\ 22 \end{matrix} \right\} (\lambda^2)^2 \\ &= \left\{ \begin{matrix} i \\ 00 \end{matrix} \right\} (\lambda^0)^2 + 2 \left\{ \begin{matrix} i \\ 01 \end{matrix} \right\} \lambda^0 \lambda^1 + 2 \left\{ \begin{matrix} i \\ 02 \end{matrix} \right\} \lambda^0 \lambda^2 \\ &+ \left\{ \begin{matrix} i \\ 11 \end{matrix} \right\} (\lambda^1)^2 + 2 \left\{ \begin{matrix} i \\ 12 \end{matrix} \right\} \lambda^1 \lambda^2 \\ &+ \left\{ \begin{matrix} i \\ 22 \end{matrix} \right\} (\lambda^2)^2 \end{aligned}$$