APPENDIX I: Geodesics in a Riemannian 3-Space

By Eq 1, the metric tensor components are as follows:

Eq 5:

$$h_{ij} = 2m(E - V)g_{ij}$$

where the g_{ij} are given in Eq 2.

The general equations for geodesics in a Riemannian 3-space are given in the next two equations. The Christoffel symbols--the expression in curly braces--will be defined later in the presentation. The summation convention--that repeated indices are summed--is used in Eq 7 and illustrated in Eq 8. The last portion of Eq 8 uses the fact that the Christoffel symbols are symmetric in the lower index pair.

Eq 6:

$$\lambda^i = \frac{dx^i}{dS}$$

Eq 7:

$$\frac{d\lambda^i}{dS} + \left\{ {i \atop \alpha\beta} \right\} \lambda^\alpha \lambda^\beta = 0$$

Expanding the portion of Eq 7 which contains the Christoffel symbols gives

Eq 8:

$$\left\{ {i \atop \alpha\beta} \right\} \lambda^\alpha \lambda^\beta = \left\{ {i \atop \alpha 0} \right\} \lambda^\alpha \lambda^0 + \left\{ {i \atop \alpha 1} \right\} \lambda^\alpha \lambda^1 + \left\{ {i \atop \alpha 2} \right\} \lambda^\alpha \lambda^2$$

$$= \left\{ {i \atop 00} \right\} (\lambda^0)^2 + \left\{ {i \atop 10} \right\} \lambda^1 \lambda^0 + \left\{ {i \atop 20} \right\} \lambda^2 \lambda^0$$
$$+ \left\{ {i \atop 01} \right\} \lambda^0 \lambda^1 + \left\{ {i \atop 11} \right\} (\lambda^1)^2 + \left\{ {i \atop 21} \right\} \lambda^2 \lambda^1$$
$$+ \left\{ {i \atop 02} \right\} \lambda^0 \lambda^2 + \left\{ {i \atop 12} \right\} \lambda^1 \lambda^2 + \left\{ {i \atop 22} \right\} (\lambda^2)^2$$

$$= \left\{ {i \atop 00} \right\} (\lambda^0)^2 + 2\left\{ {i \atop 01} \right\} \lambda^0 \lambda^1 + 2\left\{ {i \atop 02} \right\} \lambda^0 \lambda^2$$
$$+ \left\{ {i \atop 11} \right\} (\lambda^1)^2 + 2\left\{ {i \atop 12} \right\} \lambda^1 \lambda^2$$
$$+ \left\{ {i \atop 22} \right\} (\lambda^2)^2$$