computer and the numerical methods. Appendix I presents the equations which were used to implement this section in the MathCad worksheet.

The results of the calculation are shown in the first plot in the worksheet. The small "half-circle" represents mercury's orbit obtained by the standard solution described in section 4 above. The motion in time is not shown, but its points lie "right on" the orbit. The plotted points are from the Riemannian 3-space calculation. MathCad's fourth order Runge-Kutta method was used. The points seem to spiral inside the orbit of the standard solution. Since the exact solution is known from the standard treatment, this problem provides a good test for a very general implementation of the formalism of differential geometry. More accurate numerical methods seem to be required.

As a final note in this section, it should be mentioned that a calculation was undertaken to compute one of the components (R_{0101}) of the Riemann-Christoffel tensor for the space which is metrized by Eq 1. The component was found to be not identically equal to zero. Consequently, the space is not Euclidean.

6: Make the Orbit More Eccentric, & Increase m1 (the mass of the sun)

One of the most interesting features of the preceding calculation is that the orbit is computed using the same approach Einstein used in his general theory of relativity: Under Einstein's law of gravity the orbit is found as a geodesic in a Riemannian 4-space. This section is a preliminary step toward doing Einstein's calculation. It was desired to make the orbit more eccentric and to increase the mass of the sun so that mercury's perihelion shift would be easier to see. Appendix II presents the equations. First, the energy and angular momentum were changed to increase the eccentricity and, at the same time, to keep the perihelion unchanged. Then the mass of the sun was increased by a factor of 10^6, but the energy and angular momentum were adjusted so that the Newtonian orbit would not change. As shown in the first plot of the work sheet, the orbit has become much bigger. Mercury's period in this new orbit, according to the Newtonian theory, is less than 7 hours! This is clearly a relativistic problem.

7: Einstein & Schwarzschild

The second plot in the work sheet shows six revolutions in mercury's new orbit obtained by a numerical solution of the geodesic problem in a Riemannian 4-space with Schwarzschild's metric. Appendix III presents the equations. The shift of the perihelion is conspicuous, as is the fact that the plotted points are inside of the Newtonian orbit. The qualitative feature of a shifting perihelion is quite reasonable, but the numerical values are unverified, except for the fact that the calculation agrees very well with the Newtonian calculation when the parameters correspond to mercury with our sun.