where m is the reduced mass, E is the total energy, $V = -\frac{k}{r}$ is the potential energy, and where g_{ij} are the metric coefficients for spherical coordinates in Euclidean 3-space:

Eq 2:
$$g_{ij} : \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}.$$

Eq 1 was obtained from the treatment of the principal of least action in reference [2], pages 229-233. The action integral turns out to be equivalent to an expression for arc length in a space with distance defined by Eq 1. Consequently, the problem of finding a trajectory for which the action integral is stationary is equivalent to finding a geodesic in the space. Although a substantial portion of the first 200 pages of reference [2] is a prerequisite for these ideas, the treatment follows an appealing logic: Newton's laws of motion are presented in general curvilinear coordinates, then the Lagrangian equations of motion are obtained by direct calculation from Newton's laws, and then Hamilton's principle and the principal of least action are shown to be equivalent to the Lagrangian equations. With all the prerequisites, these studies were done over a period of about three years. The specific example described in this paper, along with the MathCad worksheet, were very helpful toward understanding the theory.

Regardless of whether one follows this approach or the standard solution mentioned in section 4, the following differential equations are obtained:

Eq 3:
$$\frac{d}{dt}\left(mr^2 \dot{\varphi}\right) = 0$$

Eq 4:
$$m\ddot{r} - mr\dot{\varphi}^2 + \frac{k}{r^2} = 0$$

There is a third equation which reduces to $0 = 0$ because of conservation of angular momentum: The angular momentum vector is constant and its direction is chosen as the positive *z*-axis. Then the motion occurs in the *xy*-plane so that $\theta = \pi/2$. However, the third equation is eliminated only when the initial conditions are taken into account and coordinates are chosen accordingly. In this work, the third equation is retained and the above constant value for θ is obtained numerically. Thus, the problem is that of finding a geodesic in a Riemannian 3-space, and it is necessary to solve a system of six first order differential equations. Using generalized coordinates, the problem could be posed in a Riemannian 2-space, but the present version was chosen to create greater stress for the